# كلية التقنية الالكترونية/طرابلس

## Signal & Systems part(4)

## Sheet #4

س-1- متسلسلة فوريير للإشارة x(t) معطاة بواسطة المعادلة:

$$x(t) = \sum_{n=1}^{\infty} [\frac{n}{n^2+1}] \sin(100\pi nt)$$

a. اوجد الزمن الدوري والقيمة المتوسطة للإشارة

b. ارسم طيف الاشارة

c. هل الاشارة زوجية ام فردية

d. هل الترددات التالية موجودة بطيف الاشارة

$f_1 = 150Hz,$ $f_2 = 110Hz,$ $f_3 = 640Hz,$ $f_4 = 1.6kHz$

س-2- ارسم الطيف الخطي المزدوج الجانب للإشارة:

$$f(t) = 3 + 2\cos(100t) - \cos(200t) - 0.5\sin(300t)$$

س-3- متسلسلة فوريير لإحدى الاشارات معطاة بالمعادلة:

$$x(t) = 5 + \sum_{n=1}^{\infty} \frac{20}{n\pi} \sin\frac{n\pi}{2} \cos n\frac{\pi}{2} t$$

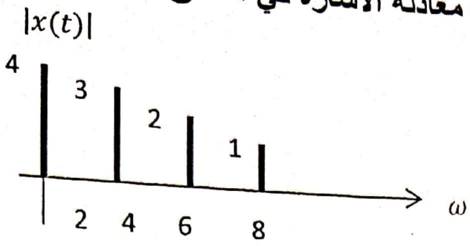
I. اوجد القيمة المتوسطة للإشارة

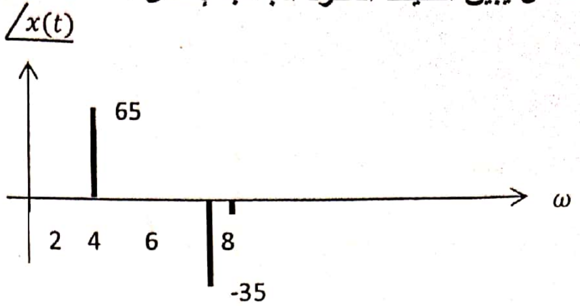
II. اوجد التردد الاساسي

III. هل الاشارة زوجية ام فردية او لا زوجية ولا فردية

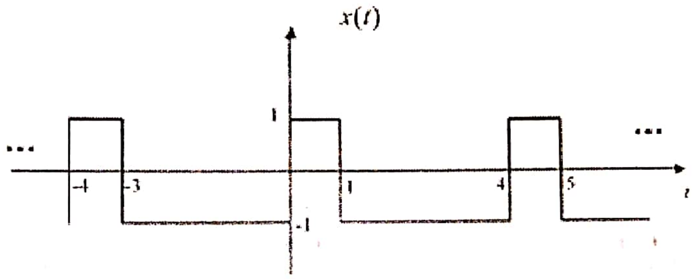
س-4- الشكل يبين الطيف المفرد الجانب لإحدى الاشارات. اكتب معادلة الاشارة في النطاق الزمني:

الشكل يبين الطيف المفرد الجانب لإحدى الاشارات. اكتب معادلة الاشارة في النطاق الزمني:




**Q(5) Q1- Compute and sketch (magnitude and phase) the Fourier series coefficients of the following signal:**




**Q(6) If the Fourier coefficients of a signal are given by:**

$$A_n = \begin{cases} \dfrac{4I_m}{n\pi} & , \ n = 1, 5, 9, \ldots\ldots \\ \dfrac{-4I_m}{n\pi} & , \ n = 3, 7, 11, \ldots \\ 0 & , \ n = even \end{cases}$$

$$B_n = 0 \qquad , \qquad A_0 = 0$$

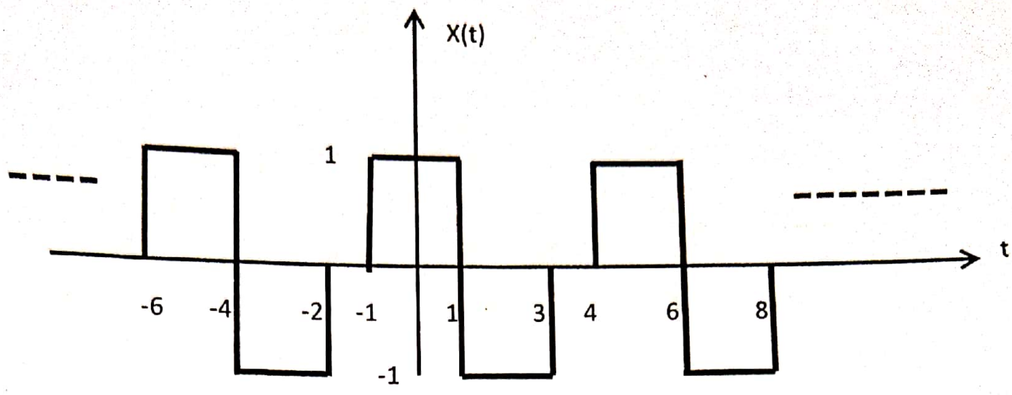
a. **Calculate the DC- value of the signal**

b. **Sketch the DSS of the signal**

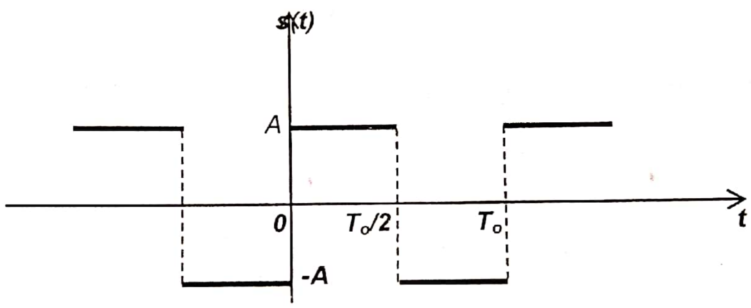
**Q(7) - Sketch the line spectrum of the following signal**

$$s(t) = 2 + 6\cos\left(2\pi 10t + 30^\circ\right) + 3\sin\left(2\pi 30t\right) - 4\cos\left(2\pi 40t\right)$$

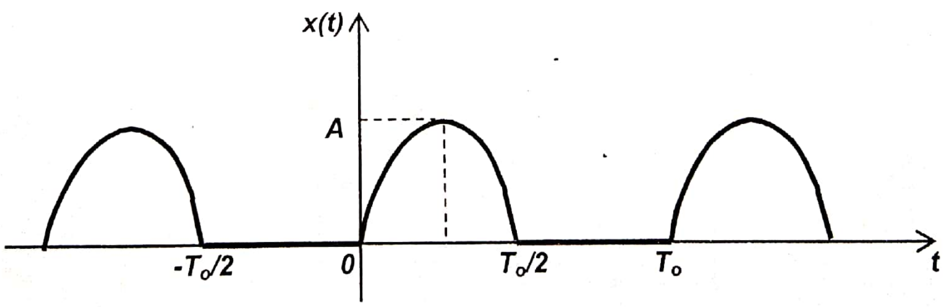
**Q(8) Find the Fourier expansion of the shown signal. [n=1, 2, 3, 4]**



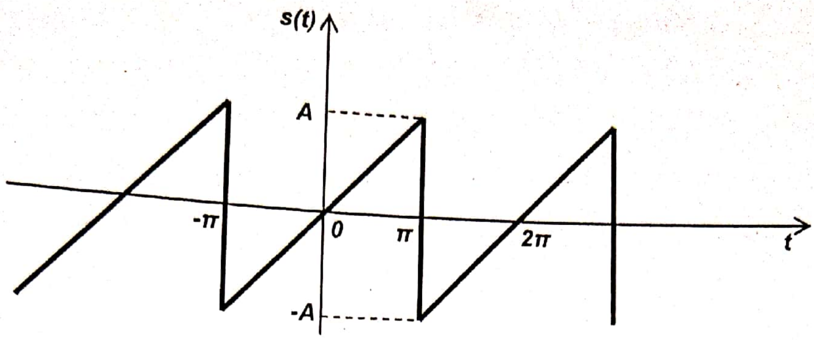

**Q(9) Find the Fourier expansion of the shown signal**




**Q(10)** Find the Fourier expansion of the shown signal



**Q(11)** Find the Fourier expansion of the shown signal




**Q(12) Calculate the average power of the signal** $s(t) = 4\sin 50\pi t$ **, using:**

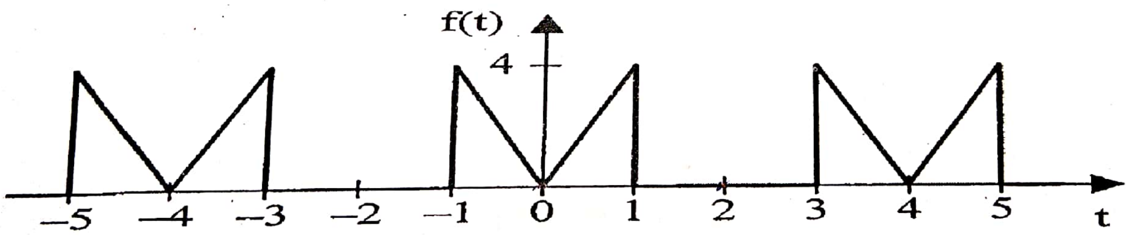
a. **The integration direct method.**
b. **The Parseval's theorem.**

**Q(13) If** $f(t) = \frac{2}{T}t$ **for** $-\frac{T}{2} < t < \frac{T}{2}$ **and** $f(t) = f(t+T)$ **has a Fourier series given by:**

$$f(t) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{2\pi nt}{T}$$

Calculate the normalized power for this signal contained in the first 3 components

Q(14) Find the Fourier series of the shown signal.



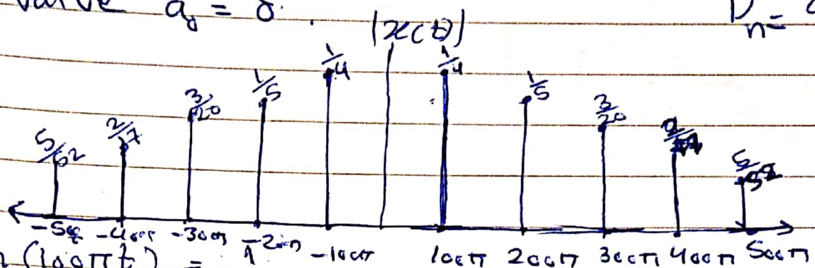
Sheet#4 (Fourier Series)

Q1:- $x(t)=\sum_{n=1}^{\infty}\left[\frac{n}{n^2+1}\right]\sin(100\pi n t)$

(a) $T=\frac{2\pi}{\omega}=\frac{2\pi}{100\pi}=\frac{1}{50}$ (sec) , $f=50$ Hz

Average value $a_0=0$. $D_n=\frac{a_n-jb_n}{2}=\frac{-jb_n}{2}$

(b)




$b_1=\boxed{\frac{1}{1+1}}\sin(100\pi t)=\frac{1}{2}$

$b_2=\frac{2}{4+1}=\frac{2}{5}$

$b_3=\frac{3}{9+1}=\frac{3}{10}$

$b_4=\frac{4}{16+1}=\frac{4}{17}$

$b_5=\frac{5}{25+1}=\frac{5}{26}$

(c) As only $\sin(n\omega t)$ coeffecient "$b_n$" is present → odd function

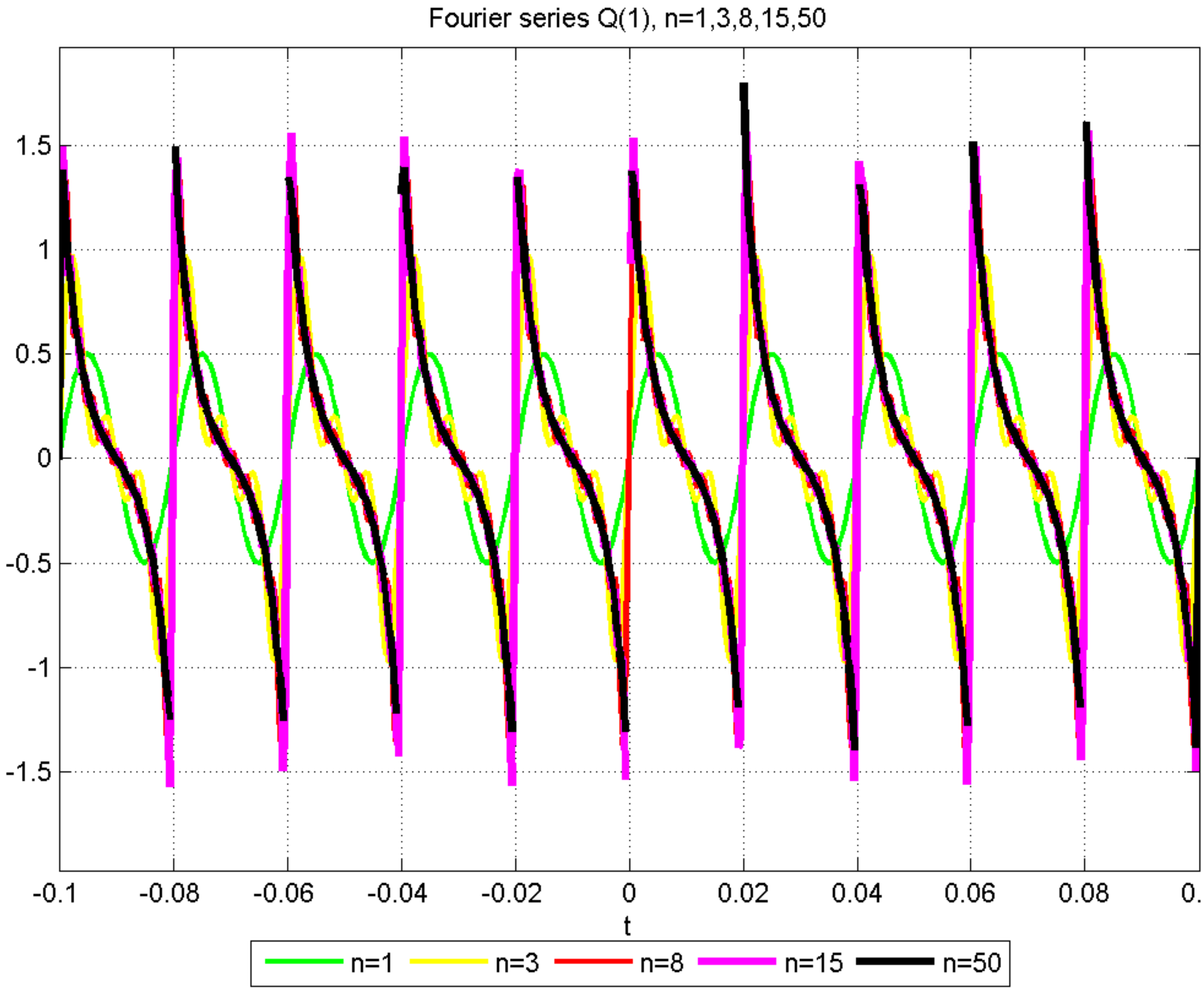
(d) The first frequency present is 50 Hz and it goes by $n\times 50$

$f_1=100\pi$ Hz ⇒ not present

$f_2=150$ Hz ⇒ present

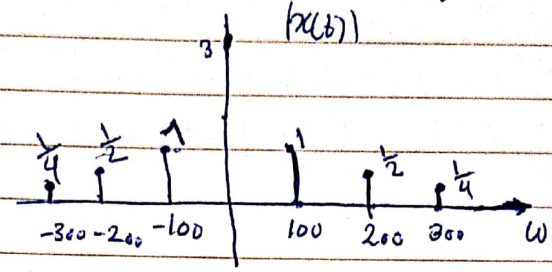
$f_3=650$ Hz ⇒ present

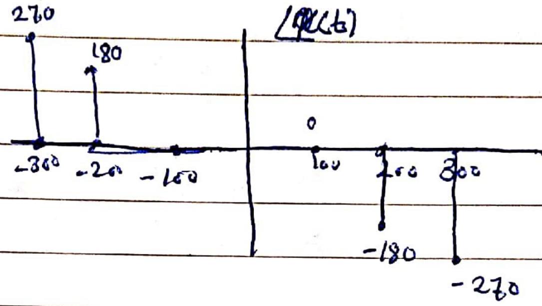
$f_4=1.6\pi$ KHz ⇒ not present


Fourier series Q(1), n=1,3,8,15,50
1.5
1
0.5
0
-0.5
-1
-1.5
-0.1
-0.08
-0.06
-0.04
-0.02
0
0.02
0.04
0.06
0.08
0.1
t
n=1
n=3
n=8
n=15
n=50

Q2:- $f(t) = 3 + 2\cos(100t) - \cos(200t) - 0.5\sin(300t)$

$f(t) = 3 + 2\cos(100t) + \cos(200t - 180^\circ) + 0.5\cos(300t - 270^\circ)$




Q3 :- $x(t) = 5 + \sum_{n=1}^{\infty} \frac{20}{n\pi} \sin\frac{n\pi}{2} \cos\frac{n\pi}{2} t$

① $a_0 = 5$ , $\omega = \frac{\pi}{2}$

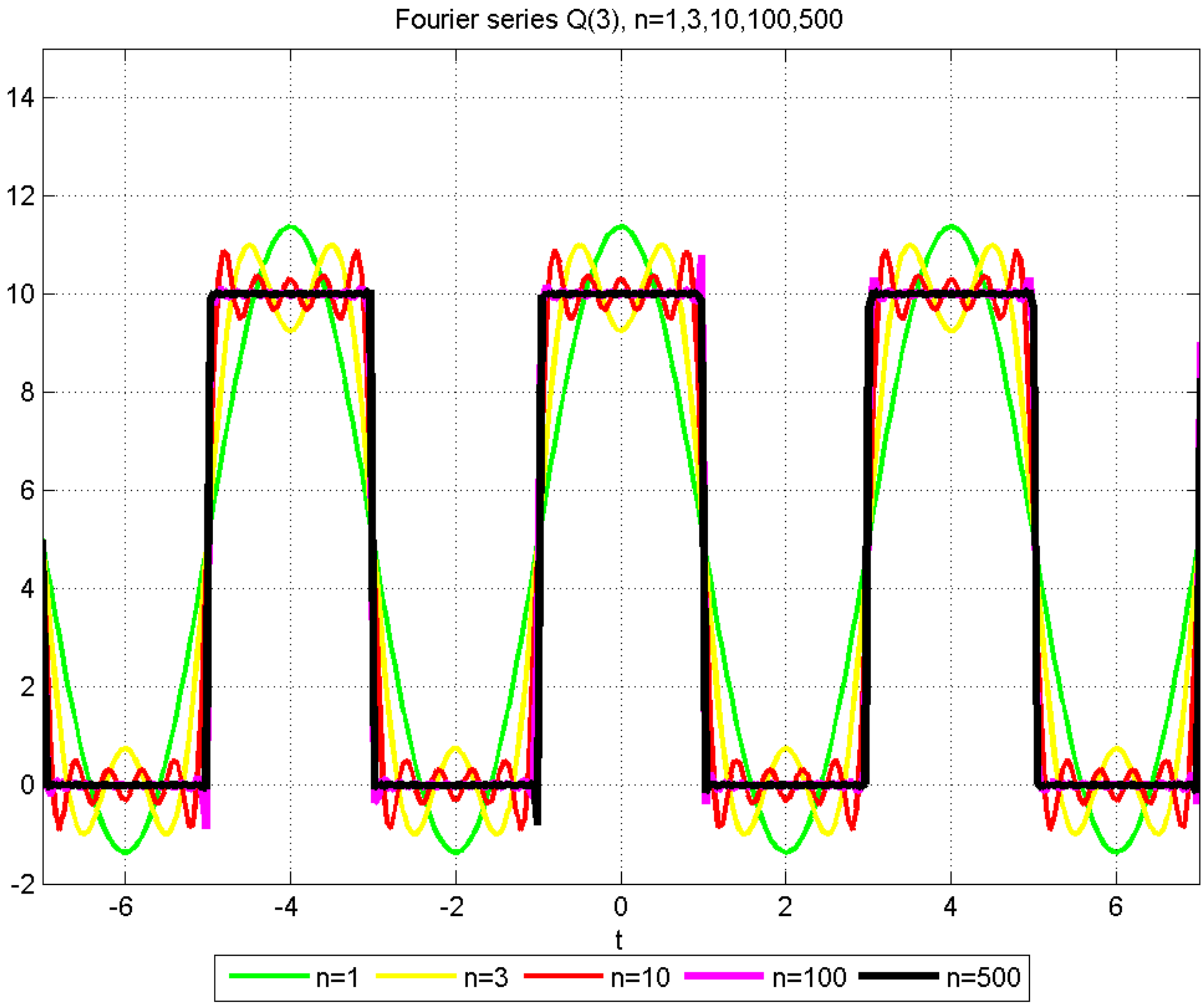
② $f = \frac{\omega}{2\pi} = \frac{\frac{\pi}{2}}{2\pi} = \frac{1}{4}$ Hz

③ The signal is even as $a_0$ and $a_n$ only are present

Q4: $x(t) = 4 + 3\cos(4t+65) + 2\cos(6t) + \cos(8t-35)$

Q5:- The signal is neither even nor odd $\Rightarrow$ all $a_0, a_1, a_2$ are present

$T_0 = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$



Fourier series Q(3), n=1,3,10,100,500
14
12
10
8
6
4
2
0
-2
-6
-4
-2
0
2
4
6
t
n=1
n=3
n=10
n=100
n=500

$$a_0 = \frac{1\times 1 - 3\times 1}{4} = \frac{-2}{4} = \frac{-2}{4} = \frac{-1}{2}$$

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ -1, & 1 \le t \le 4 \end{cases}$$

$$a_0 = \frac{1}{4}\int_0^1 dt + \frac{1}{4}\int_1^4 dt$$

$$a_0 = \frac{1}{4}\left[t\right]_0^1 + \frac{1}{4}\left[t\right]_1^4 = \frac{1}{4}[1-0] - \frac{1}{4}[4-1] = \frac{1}{4} - \frac{3}{4} = \frac{-1}{2}$$

$$a_n = \frac{2}{4}\int_0^1 \cos n\omega t\,.dt - \frac{1}{2}\int_1^4 \cos n\omega t\,.dt$$

$$a_n = \frac{1}{2\times n\omega}\left[\sin(n\omega t)\right]_0^1 - \frac{1}{2n\omega}\left[\sin(n\omega t)\right]_1^4$$

$$a_n = \frac{1}{2n\omega}\left[\sin(n\omega) - \cancel{\sin(0)}^{\,0}\right] - \frac{1}{2n\omega}\left[\sin(4n\omega) - \sin(n\omega)\right]$$

$$a_n = \frac{1}{2n\omega}\left[2\sin(n\omega) - \sin 4(n\omega)\right]$$

$$a_n = \frac{1}{2n\times\frac{\pi}{2}}\left[2\sin\left(n\times\frac{\pi}{2}\right) - \sin\left(4n\times\frac{\pi}{2}\right)\right] = \frac{1}{n\pi}\left[2\sin\left(n\times\frac{\pi}{2}\right) - \cancel{\sin(2n\pi)}^{\text{zero}}\right]$$

$$a_n = \frac{1}{n\pi}\left[2\sin\left(\frac{n\pi}{2}\right)\right]$$

$a_1 = \frac{2}{\pi}$

For $n =$ even $\Rightarrow a_n = 0$

For $n = 1, 5, 9, \ldots \Rightarrow a_n = \frac{2}{n\pi}$

$a_2 = 0$

$a_3 = \frac{-2}{3\pi}$

For $n = 3, 7, \ldots \Rightarrow a_n = \frac{-2}{n\pi}$

$a_4 = 0$, $a_5 = \frac{2}{5\pi}$

$$b_n = \frac{1}{2}\int_0^1 \sin(n\omega t)\,.dt - \frac{1}{2}\int_1^4 \cos(n\omega t)\,.dt$$

$$b_n = \frac{1}{2n\omega}\left[-\cos(n\omega t)\right]_0^1 + \frac{1}{2n\omega}\left[\cos(n\omega t)\right]_1^4$$

$$b_n = \frac{1}{2n\omega}\left[-\cos(n\omega) + \cos(0)\right] + \frac{1}{2n\omega}\left[\cos(4n\omega) - \cos(n\omega)\right]$$

$$b_n = \frac{1}{2n\omega}\left[-2\cos(n\omega) + 1 + \cos(4n\omega)\right]$$

$$b_n = \frac{1}{2n\frac{\pi}{2}}\left[-2\cos\left(n\times\frac{\pi}{2}\right) + 1 + \cos\left(4n\times\frac{\pi}{2}\right)\right]$$

$$b_n = \frac{1}{n\pi}\left[-2\cos\left(\frac{n\pi}{2}\right) + 2\right]$$

for $n$ odd $\Rightarrow b_n = \frac{2}{n\pi}$

For $n = 2, 6, 10 \Rightarrow b_n = \frac{4}{n\pi}$

For $n = 4, 8 \Rightarrow b_n = 0$

$b_1 = \frac{2}{\pi}$

$b_2 = \frac{4}{2\pi} = \frac{2}{\pi}$

$b_3 = \frac{2}{3\pi}$

$b_4 = 0$ , $b_5 = \frac{2}{5\pi}$

$C_1 = \sqrt{(a_1)^2 + (b_1)^2} = \sqrt{\left(\frac{2}{\pi}\right)^2 + \left(\frac{2}{\pi}\right)^2} = \frac{2\sqrt{2}}{\pi}$ , $\theta_1 = \tan^{-1}\frac{\frac{2}{\pi}}{\frac{2}{\pi}} = 45^\circ$

$C_2 = \sqrt{\left(\frac{2}{\pi}\right)^2} = \frac{2}{\pi}$ , $\theta_2 = \tan^{-1}\left(\frac{\frac{2}{\pi}}{0}\right) = 90^\circ$

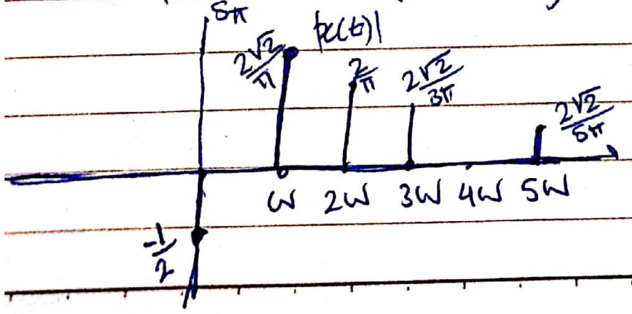
$C_3 = \sqrt{\left(\frac{-2}{3\pi}\right)^2 + \left(\frac{2}{3\pi}\right)^2} = \frac{2\sqrt{2}}{3\pi}$ , $\theta = \tan^{-1}\left(\frac{\frac{2}{3\pi}}{\frac{-2}{3\pi}}\right) = 135^\circ$

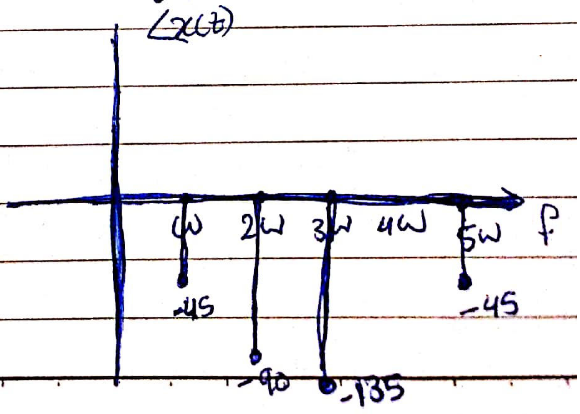
$C_4 = 0$

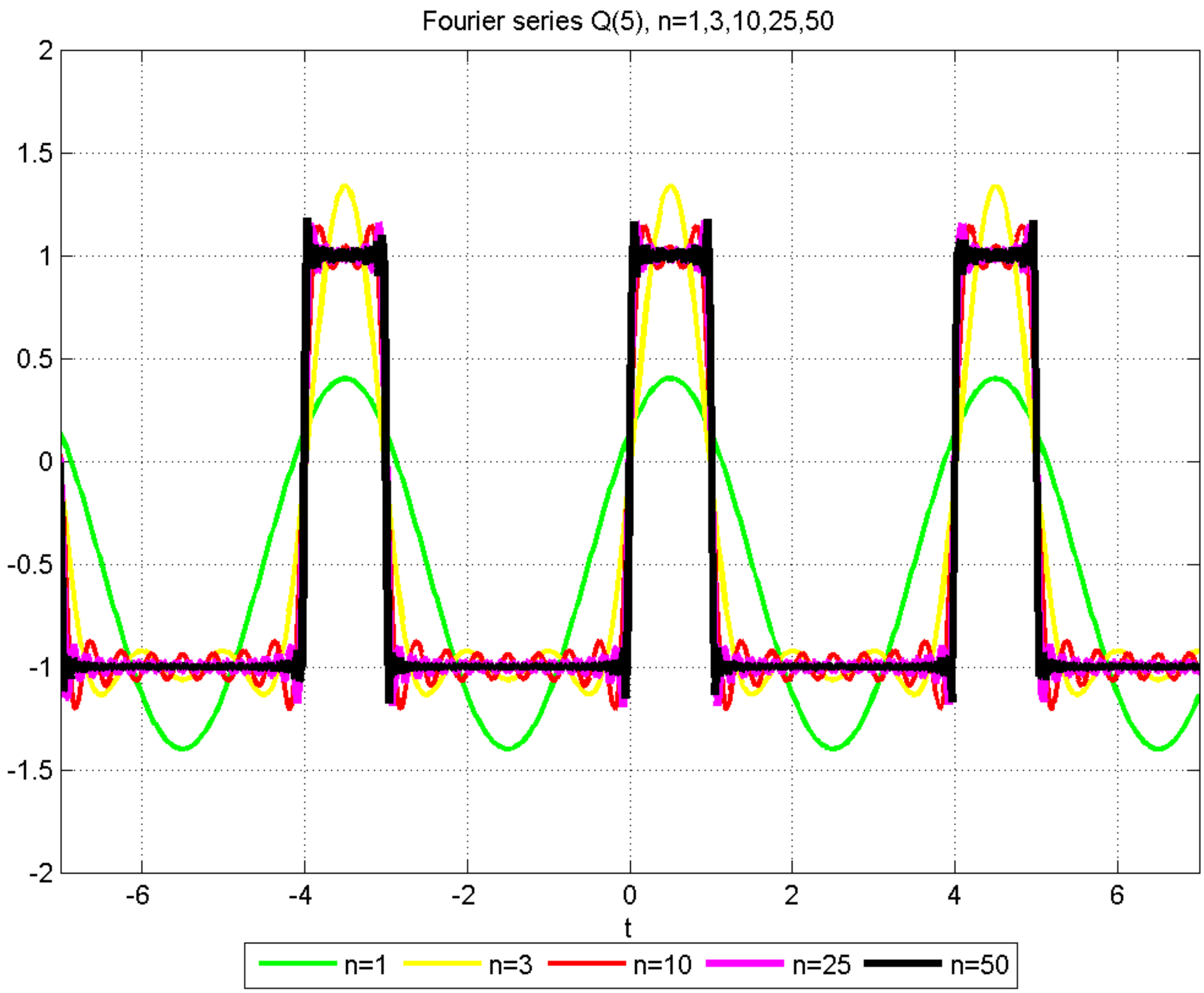
$C_5 = \sqrt{\left(\frac{2}{5\pi}\right)^2 + \left(\frac{2}{5\pi}\right)^2} = \frac{2\sqrt{2}}{5\pi}$ , $\theta = \tan^{-1}\left(\frac{\frac{2}{5\pi}}{\frac{2}{5\pi}}\right) = 45^\circ$

$$x(t) = \frac{-1}{2} + \frac{2\sqrt{2}}{\pi}\cos(\omega t - 45^\circ) + \frac{2}{\pi}\cos(2\omega t - 90^\circ) + \frac{2\sqrt{2}}{3\pi}\cos(3\omega t - 135^\circ)$$

$$+ \frac{2\sqrt{2}}{5\pi}\cos(5\omega t - 45^\circ)$$




Fourier series Q(5), n=1,3,10,25,50
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-6
-4
-2
0
2
4
6
t
n=1
n=3
n=10
n=25
n=50

Q 6 :-

$$a_n = \begin{cases} \frac{4I_m}{n\pi}, & n = 1, 5, 9, \ldots \\ \frac{-4I_m}{n\pi}, & n = 3, 7, \ldots \\ 0, & n = \text{even} \end{cases} \qquad b_n = 0, \quad a_0 = 0$$

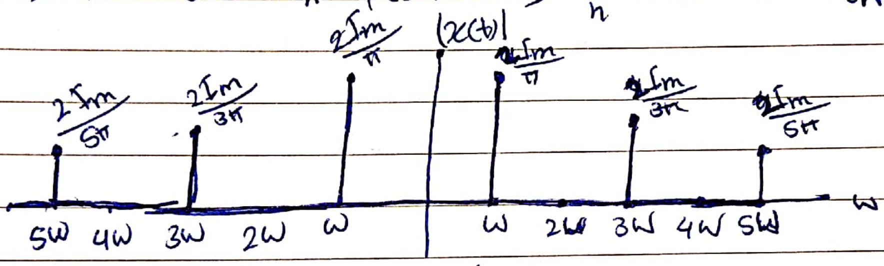
(a) DC value $= 0$ $\qquad a_1 = \frac{4I_m}{\pi}, \; a_2 = 0$

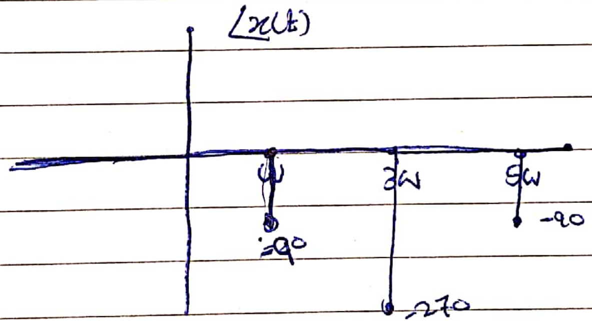
(b) DSS $\Rightarrow$ $\qquad a_3 = \frac{-4I_m}{3\pi}, \; a_4 = 0, \; a_5 = \frac{4I_m}{5\pi}$

$C_n = |a_n| \Rightarrow C_1 = \frac{4I_m}{\pi}, \; C_2 = 0, \; C_3 = \frac{4I_m}{3\pi}, \; C_4 = 0, \; C_5 = \frac{4I_m}{5\pi}$

$\theta_1 = \tan^{-1} \frac{4I_m/\pi}{0} = 90°, \; \theta_3 = 270°, \; \theta_5 = 90°$

$\theta_n \Rightarrow 90°$ for $a_n$ positive $\Rightarrow \theta_n = 270°$ for $a_n \Rightarrow$ negative.

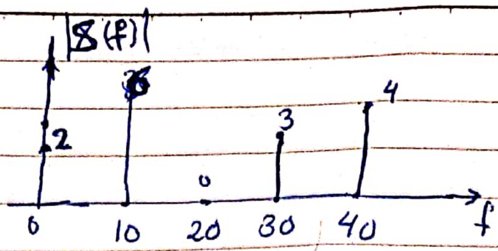


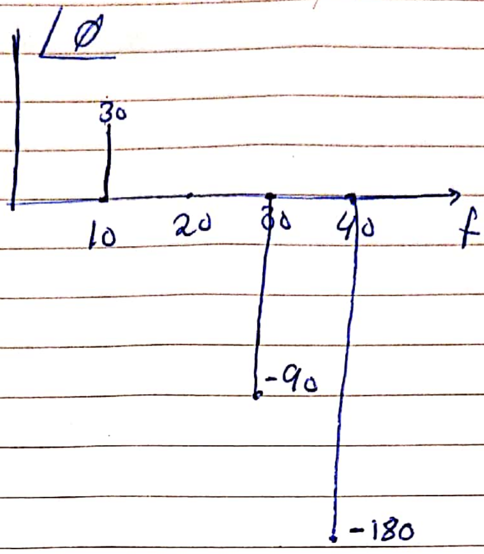




$$f(t) = \frac{4I_m}{\pi} \cos(\omega t - 90°) + 0 + \frac{4I_m}{3\pi} \cos(\omega t - 270°) + 0 + \frac{4I_m}{5\pi} \cos(\omega t - 90°)$$

Q7 :- $s(t) = 2 + 6\cos(2\pi 10 t + 30°) + 3\sin(2\pi 30 t) - 4\cos(2\pi 40 t)$

$$s(t) = 2 + 6\cos(2\pi 10 t + 30°) + 3\cos(2\pi 30 t - 90°) + 4\cos(2\pi 40 t - 180°)$$




Q8:- $a_0 = \frac{1}{T}\left[\int_{-1}^{1} 1.dt + \int_{1}^{3} 1.dt\right]$ , $T=5$ , $\omega = \frac{2\pi}{T} = \frac{2\pi}{5}$

$a_0 = \frac{1}{5}\left[ t\Big|_{-1}^{1} - t\Big|_{1}^{3} \right] = \frac{1}{5}\left[1-(-1) - (3-1)\right] = \frac{1}{5}(2-2) = 0$

It is obvious that the area ~~of~~ above the t axis equals the area down of the axis $\Rightarrow a_0 = 0$

$a_n = \frac{2}{5}\left[\int_{-1}^{1} \cos(n\omega t).dt - \int_{1}^{3} \cos(n\omega t).dt\right]$

$a_n = \frac{2}{5}\left[ \frac{\sin(n\omega t)}{n\omega}\Big|_{-1}^{1} - \frac{\sin(n\omega t)}{n\omega}\Big|_{1}^{3} \right]$

$a_n = \frac{\cancel{2}}{\cancel{5}n \times \frac{\cancel{2}\pi}{\cancel{5}}}\left[\sin(n\omega) - \sin(-n\omega) - \sin(3n\omega) + \sin(n\omega)\right]$

$$a_n = \frac{1}{n\pi}\left[\sin(n\omega) + \sin(n\omega) - \sin(3n\omega) + \sin(n\omega)\right]$$

$$a_n = \frac{1}{n\pi}\left[3\sin(n\omega) - \sin(3n\omega)\right]$$

$$a_n = \frac{1}{n\pi}\left[3\sin\left(\frac{2n\pi}{5}\right) - \sin\left(\frac{6n\pi}{5}\right)\right]$$

$$a_1 = \frac{1}{\pi}\left[3\sin\left(\frac{2\pi}{5}\right) - \sin\left(\frac{6\pi}{5}\right)\right] = 1.0953$$

$$a_2 = \frac{1}{2\pi}\left[3\sin\left(\frac{4\pi}{5}\right) - \sin\left(\frac{12\pi}{5}\right)\right] = 0.1293$$

$$a_3 = \frac{1}{3\pi}\left[3\sin\left(\frac{6\pi}{5}\right) - \sin\left(\frac{18\pi}{5}\right)\right] = -0.0862$$

$$a_4 = \frac{1}{4\pi}\left[3\sin\left(\frac{8\pi}{5}\right) - \sin\left(\frac{24\pi}{5}\right)\right] = -0.2738$$

$$b_n = \frac{2}{5}\left[\int_{-1}^{1}\sin(n\omega t)\,.dt - \int_{1}^{3}\sin(n\omega t)\,.dt\right]$$

$$b_n = \frac{2}{5n\omega}\left[-\cos(n\omega t)\Big|_{-1}^{1} + \cos(n\omega t)\Big|_{1}^{3}\right]$$

$$b_n = \frac{1}{n\pi}\left[-\cos(n\omega) + \cos(-n\omega) + \cos(3n\omega) - \cos(n\omega)\right]$$

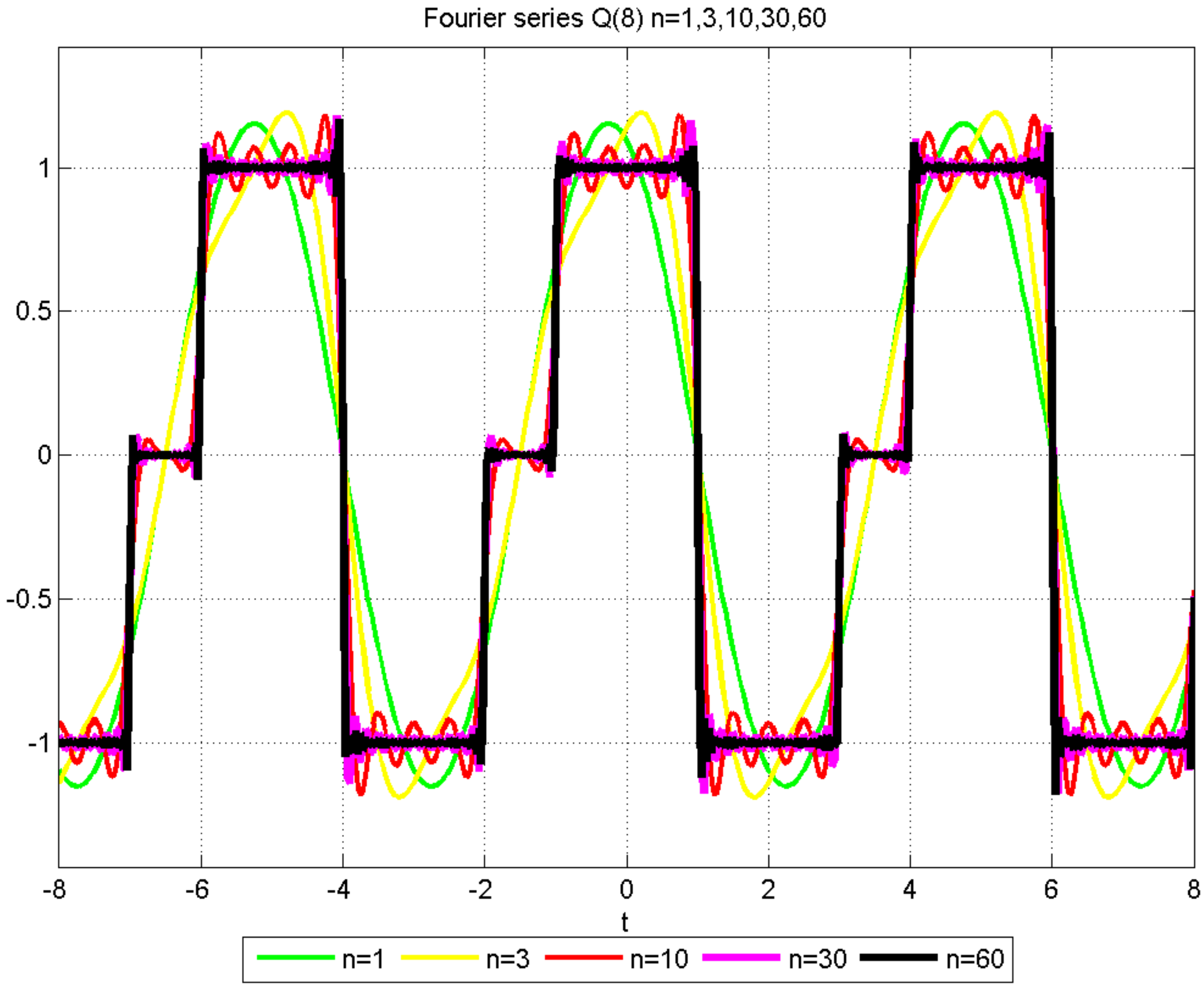
$$b_n = \frac{1}{n\pi}\left[\cancel{-\cos(n\omega) + \cos(n\omega)} + \cos(3n\omega) - \cancel{\cos(n\omega)}\right]$$

$$b_n = \frac{1}{n\pi}\left[\cos(3n\omega) - \cos(n\omega)\right]$$

$$b_n = \frac{1}{n\pi}\left[\cos\left(\frac{6n\pi}{5}\right) - \cos\left(\frac{2n\pi}{5}\right)\right]$$

$$b_1 = \frac{1}{\pi}\left[\cos\left(\frac{6\pi}{5}\right) - \cos\left(\frac{2\pi}{5}\right)\right] = -0.356$$

$$b_2 = \frac{1}{2\pi}\left[\cos\left(\frac{12\pi}{5}\right) - \cos\left(\frac{4\pi}{5}\right)\right] = 0.178$$

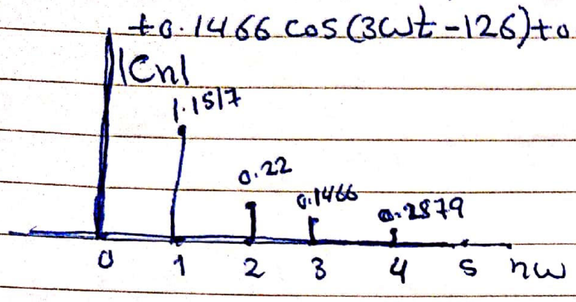

Fourier series Q(8) n=1,3,10,30,60
1
0.5
0
-0.5
-1
-8
-6
-4
-2
0
2
4
6
8
t
n=1
n=3
n=10
n=30
n=60

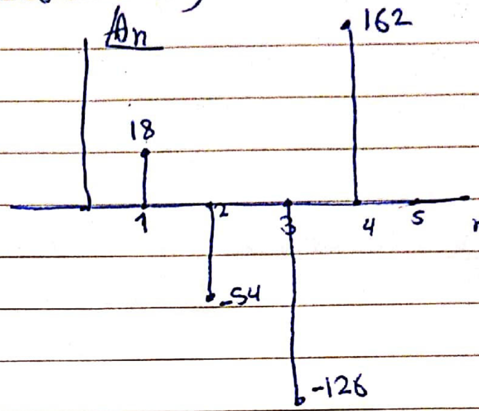
$$b_3 = \frac{1}{3\pi}\left[\cos\left(\frac{18\pi}{5}\right) - \cos\left(\frac{6\pi}{5}\right)\right] = 0.1186$$

$$b_4 = \frac{1}{4\pi}\left[\cos\left(\frac{24\pi}{5}\right) - \cos\left(\frac{8\pi}{5}\right)\right] = -0.089$$

| $n$ | $C_n$ | $-\tan^{-1}\left(\frac{b_n}{a_n}\right)$ |
|---|---|---|
| 1 | 1.1517 | 18 |
| 2 | 0.22 | -54 |
| 3 | 0.1466 | -126 |
| 4 | 0.2879 | 162 |
| 5 | 0 | 0 |

$x(t) = 1.1517\cos(\omega t + 18) + 0.22\cos(2\omega t - 54)$
$+ 0.1466\cos(3\omega t - 126) + 0.2879\cos(4\omega t + 162)$




Q9:- The signal is odd $\Rightarrow a_0, a_n = 0$, The signal has half wave symmetry

$$b_n = \frac{2A}{T_0}\left[\int_0^{\frac{T_0}{2}} \sin(n\omega t)\,dt - \int_{\frac{T_0}{2}}^{T_0} \sin(n\omega t)\,dt\right]$$

$$b_n = \frac{2A}{T_0}\left[\left.\frac{-\cos(n\omega t)}{n\omega}\right|_0^{\frac{T_0}{2}} + \left.\frac{\cos(n\omega t)}{n\omega}\right|_{\frac{T_0}{2}}^{T_0}\right]$$

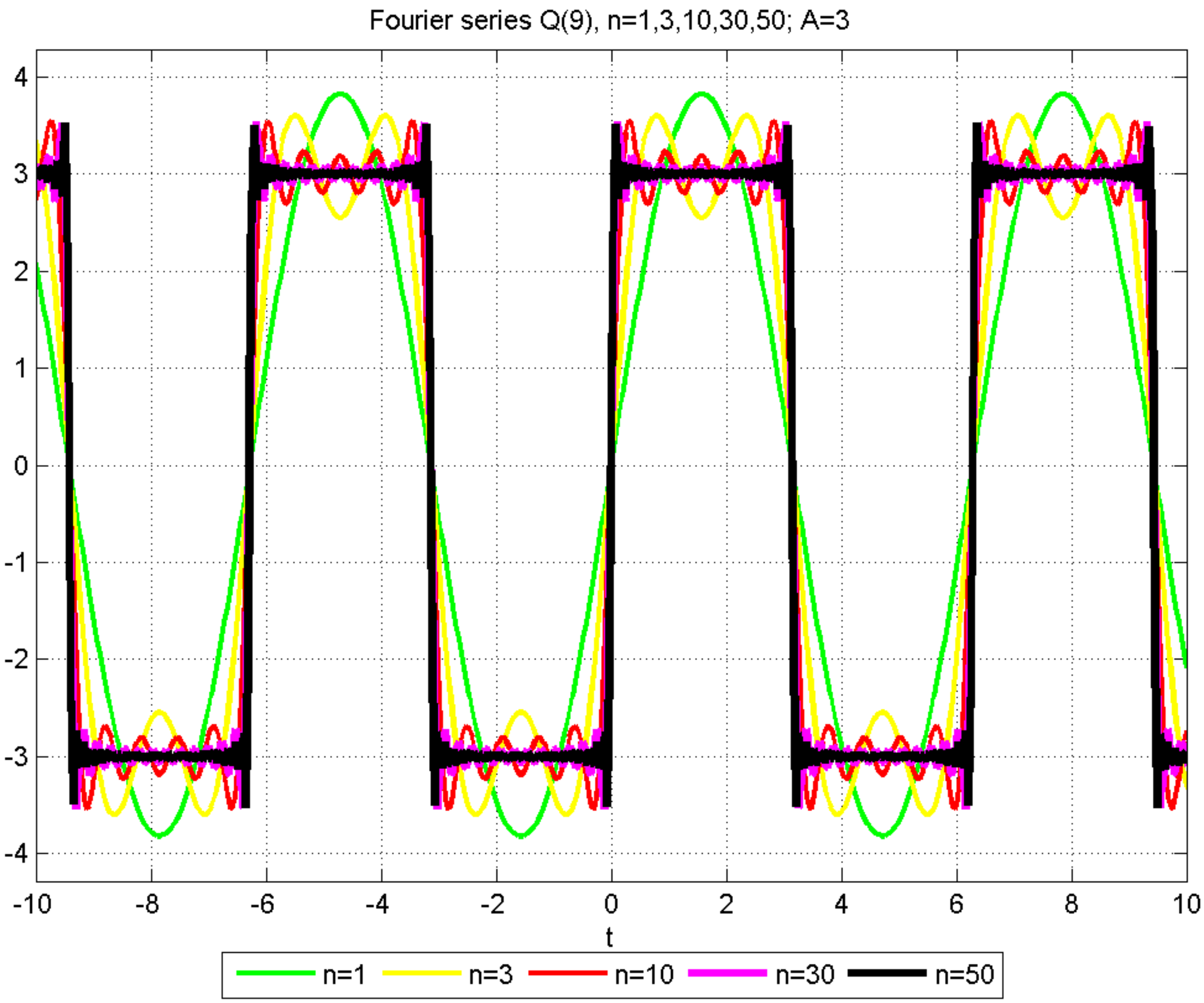
$$b_n = \frac{2A}{nT_0\omega}\left[-\cos\left(n\omega\frac{T_0}{2}\right) + \cos(0) + \cos(n\omega T_0) - \cos\left(n\omega\frac{T_0}{2}\right)\right]$$

$\omega T_0 = 2\pi$ , $\omega \times \frac{T_0}{2} = \pi$

$$b_n = \frac{\cancel{2}A}{n\cancel{2}\pi}\left[-\cos(n\pi) + 1 + \cos(2n\pi) - \cos(n\pi)\right]$$

$$b_n = \frac{A}{n\pi}\left[1 + \cos(2n\pi) - 2\cos(n\pi)\right]$$

note that :- $\cos(2n\pi) = 1$ , $\cos(n\pi) = \begin{cases} 1, & \text{for } (n) \text{ even} \\ -1, & \text{for } (n) \text{ odd} \end{cases}$


Fourier series Q(9), n=1,3,10,30,50; A=3
4
3
2
1
0
-1
-2
-3
-4
-10
-8
-6
-4
-2
0
2
4
6
8
10
t
n=1
n=3
n=10
n=30
n=50

$$b_n = \frac{A}{n\pi}\left[2 - 2\cos(n\pi)\right]$$

For $n$(even) $\Rightarrow$ $b_n = \frac{A}{n\pi}\left[2-2\right] = 0$

$b_n$ for $n$(odd) $\Rightarrow b_n = \frac{A}{n\pi}\left[2+2\right] = \frac{4A}{n\pi}$

$$x(t) = \frac{4A}{\pi}\sum_{n=odd}^{\infty} \frac{\sin(n\omega t)}{n}$$

$$x(t) = \frac{4A}{\pi}\sin(\omega t) + \frac{4A}{3\pi}\sin(3\omega t) + \frac{4A}{5\pi}\sin(5\omega t) + \frac{4A}{7\pi}\sin(7\omega t)$$

Q10 :- $T_0 = 2\pi$ , $\omega T_0 = 2\pi$ , $\omega = 1$ , $\frac{T}{2} = \pi$

$$x(t) = \begin{cases} A\sin\omega_0 t, & 0 \leq t \leq \frac{T_0}{2} \\ 0 & , \frac{T_0}{2} \leq t \leq T_0 \end{cases}$$

$$a_0 = \frac{A}{2\pi}\int_0^{\pi} \sin\omega_0 t \, dt$$

$$a_0 = \frac{A}{2\pi} \left. -\cos t \right|_0^{\pi} = \frac{A}{2\pi}\left[-\cos(\pi) + \cos(0)\right] = \frac{A}{2\pi}[2] = \frac{A}{\pi}$$

$$a_n = \frac{A}{\pi}\int_0^{\pi} \sin(\omega_0 t)\cos(n\omega t)\, dt$$

$$a_n = \frac{A}{\pi}\int_0^{\pi} \sin(t)\cos(nt)\, dt$$

This is on the form of $\sin(a)\cos(b) = \frac{\sin(a+b) + \sin(a-b)}{2}$

$$a_n = \frac{A}{\pi}\int_0^{\pi} \frac{\sin(1+n)t + \sin(1-n)t}{2}\, dt$$

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$$a_n = \frac{A}{2\pi}\left[\frac{-\cos(1+n)t}{(1+n)} - \frac{\cos(1-n)t}{(1-n)}\right]_0^{\pi}$$

$$a_n = \frac{A}{2\pi}\left[\frac{-\cos(1+n)\pi}{(1+n)} - \frac{\cos(1-n)\pi}{(1-n)} + \frac{\cos(0)}{(1+n)} + \frac{\cos(0)}{(1-n)}\right]$$

* $\cos(1+n)\pi = -\cos(n\pi)$
* $\cos(1-n)\pi = -\cos(n\pi)$

$$a_n = \frac{A}{2\pi}\left[\frac{\cos(n\pi)}{(1+n)} + \frac{\cos(n\pi)}{(1-n)} + \frac{\cos(0)}{(1+n)} + \frac{\cos(0)}{(1-n)}\right]$$

$$a_n = \frac{A}{2\pi}\left[\frac{(-1)^n}{(1+n)} + \frac{(-1)^n}{(1-n)} + \frac{1}{1+n} + \frac{1}{1-n}\right]$$

For $n$ even $\Rightarrow (-1)^n = 1$, for $n$ odd $(-1)^n = -1$

For $n$ even $\Rightarrow a_n = \frac{A}{2\pi}\left[\frac{1}{1+n} + \frac{1}{1-n} + \frac{1}{1+n} + \frac{1}{1-n}\right]$

$$a_n = \frac{A}{2\pi}\left[\frac{2}{1+n} + \frac{2}{1-n}\right] = \frac{A}{\pi}\left[\frac{1}{1+n} + \frac{1}{1-n}\right], n \neq 1$$

$$a_n = \frac{A}{\pi}\left[\frac{1\cancel{-n}+1\cancel{+n}}{1-n^2}\right] = \frac{2A}{\pi(1-n^2)}$$ For $n$ even and $n \neq 1$

$a_n \Rightarrow n$ odd $= 0$

For $n=1 \Rightarrow a_n = \frac{A}{\pi}\int_0^{\pi} \sin(t)\cos(t).dt = \frac{A}{2\pi}\sin^2 t\Big|_0^{\pi}$

$$a_n = \frac{A}{2\pi}\left[\sin^2(\pi) - \sin^2(0)\right] = 0.$$

For $b_n = \frac{A}{\pi}\int_0^{\pi} \sin(t)\sin(nt).dt$

$$\sin(a)\sin(b) = \frac{\cos(a-b) - \cos(a+b)}{2}$$

$$b_n = \frac{A}{2\pi}\int_0^{\pi}\left(\cos(1-n)t - \cos(1+n)t\right).dt$$

$$b_n = \frac{A}{2\pi}\left[\frac{\sin(1-n)t}{(1-n)} - \frac{\sin(1+n)t}{(1+n)}\right]_0^{\pi}, \quad n \neq 1$$

$$b_n = \frac{A}{2\pi}\left[\frac{\sin(1-n)\pi}{1-n} - \frac{\sin(1+n)\pi}{(1+n)} - \frac{\sin(0)}{(1-n)} + \frac{\sin(0)}{(1+n)}\right] \quad n \neq 1$$

$\sin(1-n)\pi = \sin(1+n)\pi = 0$

$b_n = 0$

For $n=1 \Rightarrow b_n = \frac{A}{\pi}\int_0^{\pi} \sin(t)\sin(t).dt$

$$b_n = \frac{A}{\pi}\int_0^{\pi} \sin^2(t).dt$$

$$b_n = \frac{A}{2\pi}\int_0^{\pi} (1-\cos 2t).dt$$

$$b_n = \frac{A}{2\pi}\left[t + \frac{\sin(2t)}{2}\right]_0^{\pi}$$

$$b_n = \frac{A}{2\pi}\left[\pi + \frac{\sin(2\pi)}{2} - 0 - \frac{\sin(0)}{2}\right]$$

$b_n = \frac{A}{2\cancel{\pi}} \times \cancel{\pi} = \frac{A}{2}$, For $n=1$

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin(t) + \frac{2A}{\pi}\sum_{n=\text{even}}^{\infty}\frac{1}{(1-n^2)}\cos(nt)$$

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin(t) - \frac{2A}{3\pi}\cos(2t) - \frac{2A}{15\pi}\cos(4t) - \frac{2A}{35\pi}\cos(6t)$$

$$- \frac{2A}{63\pi}\cos(8t) - \frac{2A}{99\pi}\cos(10t) + \ldots$$

$c_1 = \sqrt{(a_1)^2 + (b_1)^2} = \sqrt{0 + \left(\frac{A}{2}\right)^2} = \frac{A}{2}$, $\theta_1 = \tan^{-1}\left(\frac{A/2}{0}\right) = \frac{\pi}{2}$

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$C_2 = \sqrt{(a_2)^2 + (b_2)^2} = \sqrt{\left(\frac{-2A}{3\pi}\right)^2 + 0} = \frac{2A}{3\pi}$ $\quad \theta_2 = 0$

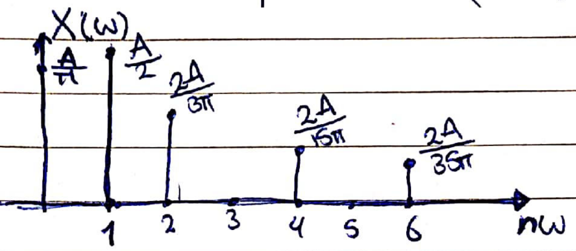
$C_3 = 0$

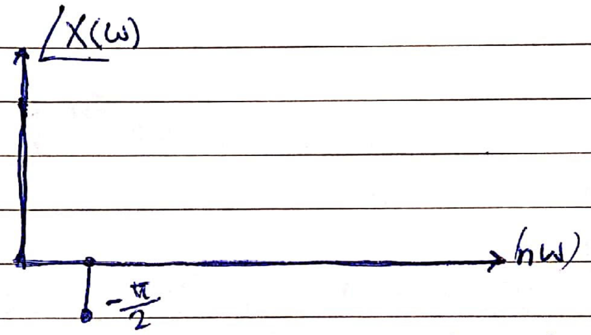
$C_4 = \sqrt{\left(\frac{-2A}{15\pi}\right)^2 + 0} = \frac{2A}{15\pi}$, $\theta_4 = 0$

$C_5 = 0$

$C_6 = \sqrt{\left(\frac{-A}{35\pi}\right)^2 + 0} = \frac{2A}{35\pi}$, $\theta_6 = 0$

The single side spectrum (SSS)




$x(t) = \frac{A}{\pi} + \frac{A}{2}\cos\left(t - \frac{\pi}{2}\right) - \frac{2A}{3\pi}\cdot\cos(2t) - \frac{2A}{15\pi}\cos(4t) - \frac{2A}{35\pi}\cos(6t)$

Also we can solve it using Exponential Fourier Series.

$D_n = \frac{A}{T}\int_0^{\pi} \sin \omega t \, e^{-nj\omega t} \, .dt$ $\qquad \omega = 1$

$D_n = \frac{A}{2\pi}\int_0^{\pi} \sin t \, . \, e^{-jnt} \, .dt$

$u = \sin(t)$ $\qquad dv = e^{-jnt}.dt$

$du = \cos t . dt$ $\qquad v = \frac{e^{-jt}}{-nj}$

$= \frac{-\sin(t)\, e^{-jnt}}{nj} + \frac{1}{nj}\int \cos t \, e^{-jnt} \, .dt$

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$$\int_0^{\pi} \sin(t)\, e^{-jnt}\, dt = \left.\frac{-\sin(t)\, e^{-jnt}}{nj}\right|_0^{\pi} + \frac{1}{nj}\int_0^{\pi} \cos(t)\, e^{-jnt}\, dt$$

second integration by parts.

$$u = \cos(t) \qquad dv = e^{-jnt}\, dt$$

$$du = -\sin(t)\, dt \qquad v = \frac{e^{-jnt}}{-jn}$$

$$\int_0^{\pi} \sin(t)\, e^{-jt}\, dt = \frac{-\sin(t)\, e^{-njt}}{nj} - \frac{1}{n^2 j^2}\cos(t)\, e^{-njt} - \frac{1}{n^2 j^2}\int_0^{\pi} \sin(t)\, e^{-njt}\, dt$$

$$-\frac{1}{j^2} \Rightarrow 1$$

$$\int_0^{\pi} \sin(t)\, e^{-njt}\, dt = \frac{-\sin(t)\, e^{-njt}}{nj} + \frac{1}{n^2}\cos(t)\, e^{-njt} + \frac{1}{n^2}\int_0^{\pi} \sin(t)\, e^{-njt}\, dt$$

$$\left(1 - \frac{1}{n^2}\right)\int_0^{\pi} \sin(t)\, e^{-njt}\, dt = \frac{-\sin(t)\, e^{-njt}}{nj} + \frac{1}{n^2}\cos(t)\, e^{-njt}$$

$$\int_0^{\pi} \sin(t)\, e^{-njt}\, dt = \frac{n^{\cancel{2}}}{n^2-1} \times \frac{1}{\cancel{n}}\left[j\sin(t)\, e^{-njt} + \frac{1}{n}\cos(t)\, e^{-njt}\right]$$

$$D_n = \frac{An}{2\pi(n^2-1)}\left[j\sin(t)\, e^{-njt} + \frac{1}{n}\cos(t)\, e^{-njt}\right]_0^{\pi}$$

$$D_n = \frac{An}{2\pi(n^2-1)}\left[\cancel{j\sin(\pi)}^{\,0}\, e^{-n\pi j} + \frac{1}{n}\cos(\pi)\, e^{-n\pi j} - \cancel{j\sin(0)}^{\,0}\, e^{0} - \frac{1}{n}\cos(0)\, e^{0}\right]$$

$$D_n = \frac{An}{2\pi(n^2-1)}\left[\frac{-1}{n}\, e^{-n\pi j} - \frac{1}{n}\right]$$

$$e^{-n\pi j} = \cos(n\pi) - j\sin(n\pi) = \cos(n\pi) = (-1)^n$$

$$D_n = \frac{nA}{(n^2-1)\,2\pi}\left[\frac{-1\times(-1)^n}{n} - \frac{1}{n}\right], \quad n \neq \pm 1$$

$$D_n = \frac{nA}{(n^2-1)\,2\pi}\left[\frac{(-1)^{n+1}}{n} - \frac{1}{n}\right], \quad n \neq \pm 1$$

For $n$ odd $\Rightarrow (-1)^{n+1} = 1 \Rightarrow D_n = 0$

For $n$ even $\Rightarrow D_n = \dfrac{nA}{(n^2-1)\,2\pi}\left[\dfrac{-1}{n} - \dfrac{1}{n}\right]$

$$D_n = \frac{-\cancel{2}\cancel{n}A}{\cancel{n}(n^2-1)\cancel{2}\pi} = \frac{-A}{(n^2-1)\pi} = \frac{A}{(1-n^2)\pi}$$

For $n=1 \Rightarrow$ ~~$D_1$~~ $D_0 = \dfrac{A}{2\pi}\displaystyle\int_0^{\pi} \sin(t)\, e^{jt}\,.dt$

$$D_1 = \frac{A}{2\pi}\int_0^{\pi} \frac{e^{jt} - e^{-jt}}{2j}\,.\,e^{-jt}\,.dt$$

$$D_1 = \frac{A}{4\pi j}\int_0^{\pi}\left(-e^{-2jt} + 1\right) dt$$

$$D_1 = \frac{A}{4\pi j}\left[t + \frac{e^{-2jt}}{2j}\right]_0^{\pi}$$

$$D_1 = \frac{A}{4\pi j}\left[\pi + \frac{e^{-2\pi j}}{2j} - 0 - \frac{e^{0}}{2j}\right]$$

$$D_1 = \frac{A}{4\pi j}\left[\pi + \frac{\cos(2\pi) - j\sin(2\pi)}{2j} - \frac{1}{2j}\right]$$

$$D_1 = \frac{A}{4\cancel{\pi} j}\left[\cancel{\pi} + \cancel{\frac{1}{2j}} - \cancel{\frac{1}{2j}}\right]$$

$$D_1 = \frac{A}{4j}$$

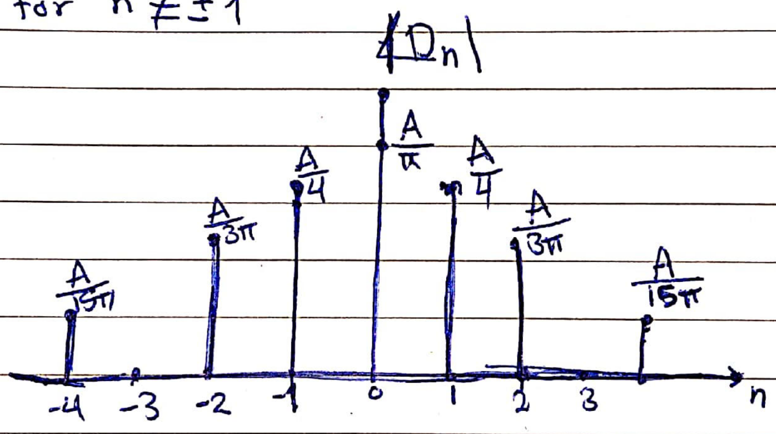
For $D_{-1}$ which is the conjugate

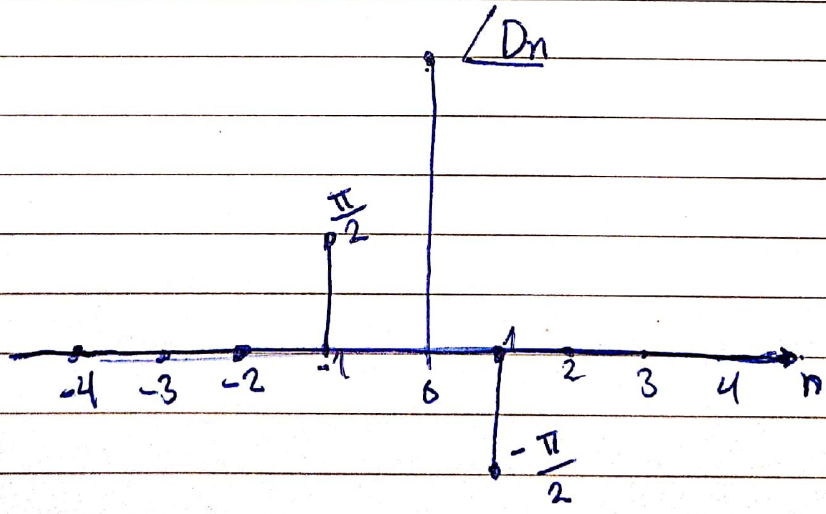
$$D_{-1} = D_1^* = \frac{-A}{4j}$$

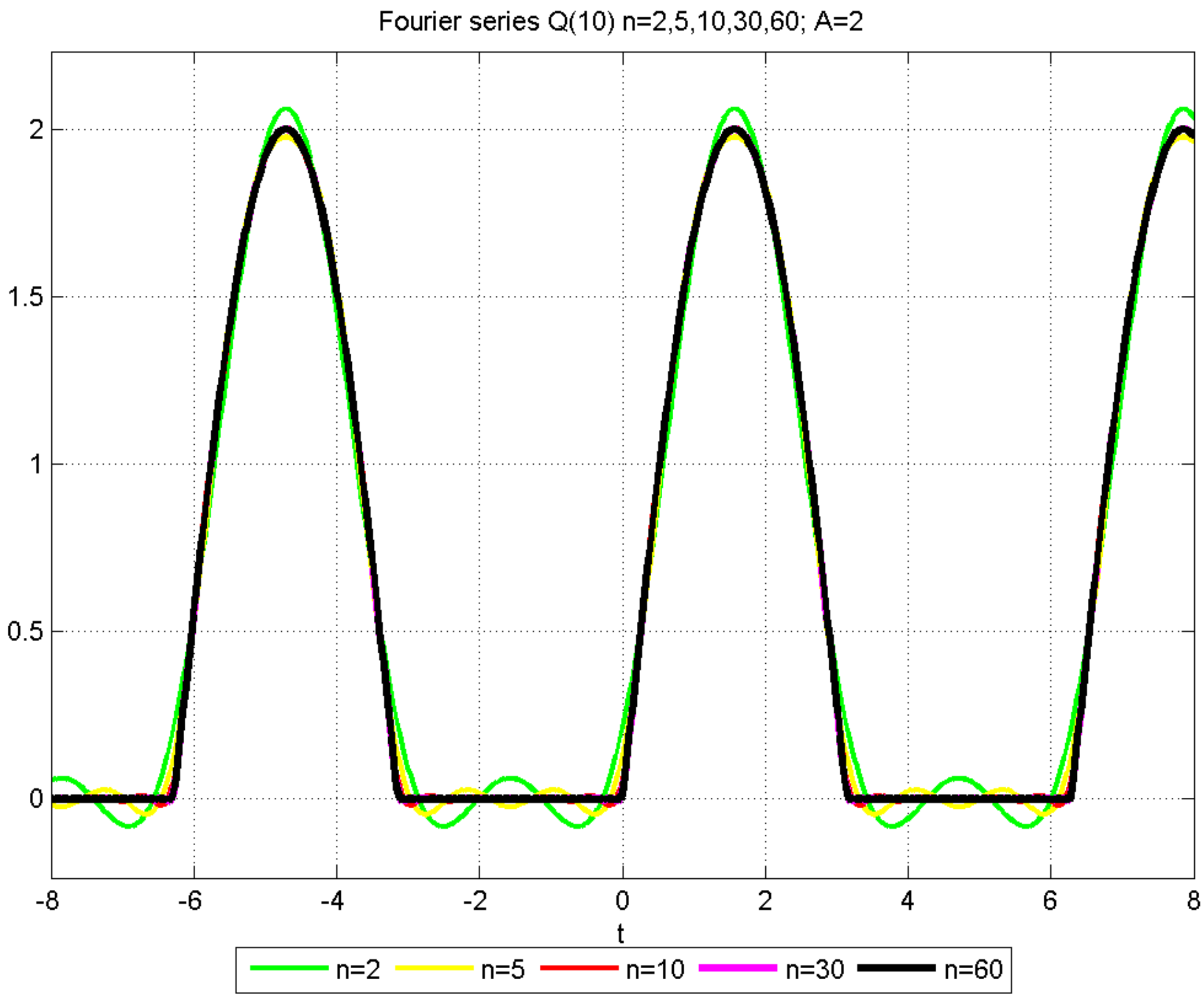
$$D_n = \begin{cases} \frac{A}{(1-n^2)\pi} & \text{for } n = 0, \pm 2, \pm 4, \pm 6, \ldots \\ 0 & \text{for } n = \pm 3, \pm 5, \pm 7, \ldots \\ -j\frac{A}{4} & \text{for } n = -1 \\ j\frac{A}{4} & \text{for } n = 1 \end{cases}$$

$$|D_n| = \begin{cases} \left|\frac{A}{(1-n^2)\pi}\right| & \text{for } n = 0, \pm 2, \pm 4, \pm 6, \ldots \\ \frac{A}{4} & \text{for } n = \pm 1 \\ 0 & \text{for } n = \pm 3, \pm 5, \pm 7, \ldots \end{cases}$$

$$\angle\theta_n = \begin{cases} \tan^{-1}\left(\frac{-\frac{A}{4}}{0}\right) = -\frac{\pi}{2} & \text{for } n = 1 \\ \tan^{-1}\left(\frac{A/4}{0}\right) = \frac{\pi}{2} & \text{for } n = -1 \\ 0 & \text{for } n \neq \pm 1 \end{cases}$$





Fourier series Q(10) n=2,5,10,30,60; A=2
2
1.5
1
0.5
0
-8
-6
-4
-2
0
2
4
6
8
t
n=2
n=5
n=10
n=30
n=60

Q11 , The Function ~~of~~ the signal is odd $\Rightarrow a_0, a_n = 0$

$T = 2\pi$ , $\omega = 1$

$$S(t) = \frac{A}{\pi} t \qquad -\pi \leq t \leq \pi$$

$$b_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} \frac{A}{\pi} t \sin(nt) . dt$$

$$b_n = \frac{A}{\pi^2} \int_{-\pi}^{\pi} t . \sin(nt) . dt$$

$$b_n = \frac{A}{\pi^2} \left[ \frac{-t\cos(nt)}{n} + \int_{-\pi}^{\pi} \frac{\cos(nt)}{n} . dt \right.$$

$u = t \qquad dv = \sin(nt)$

$du = dt \qquad v = \frac{-\cos(nt)}{n}$

$$b_n = \frac{A}{n\pi^2} \left[ -t\cos(nt) + \frac{\sin(nt)}{n} \right]_{-\pi}^{\pi}$$

$$b_n = \frac{A}{n\pi^2} \left[ -\pi\cos(n\pi) + \cancel{\frac{\sin(n\pi)}{n}}^{0} - \pi\cos(-n\pi) - \cancel{\frac{\sin(-n\pi)}{n}}^{0} \right]$$

$$b_n = \frac{A}{n\pi^{\cancel{2}}} \left[ -2\cancel{\pi} \cos(n\pi) \right]$$

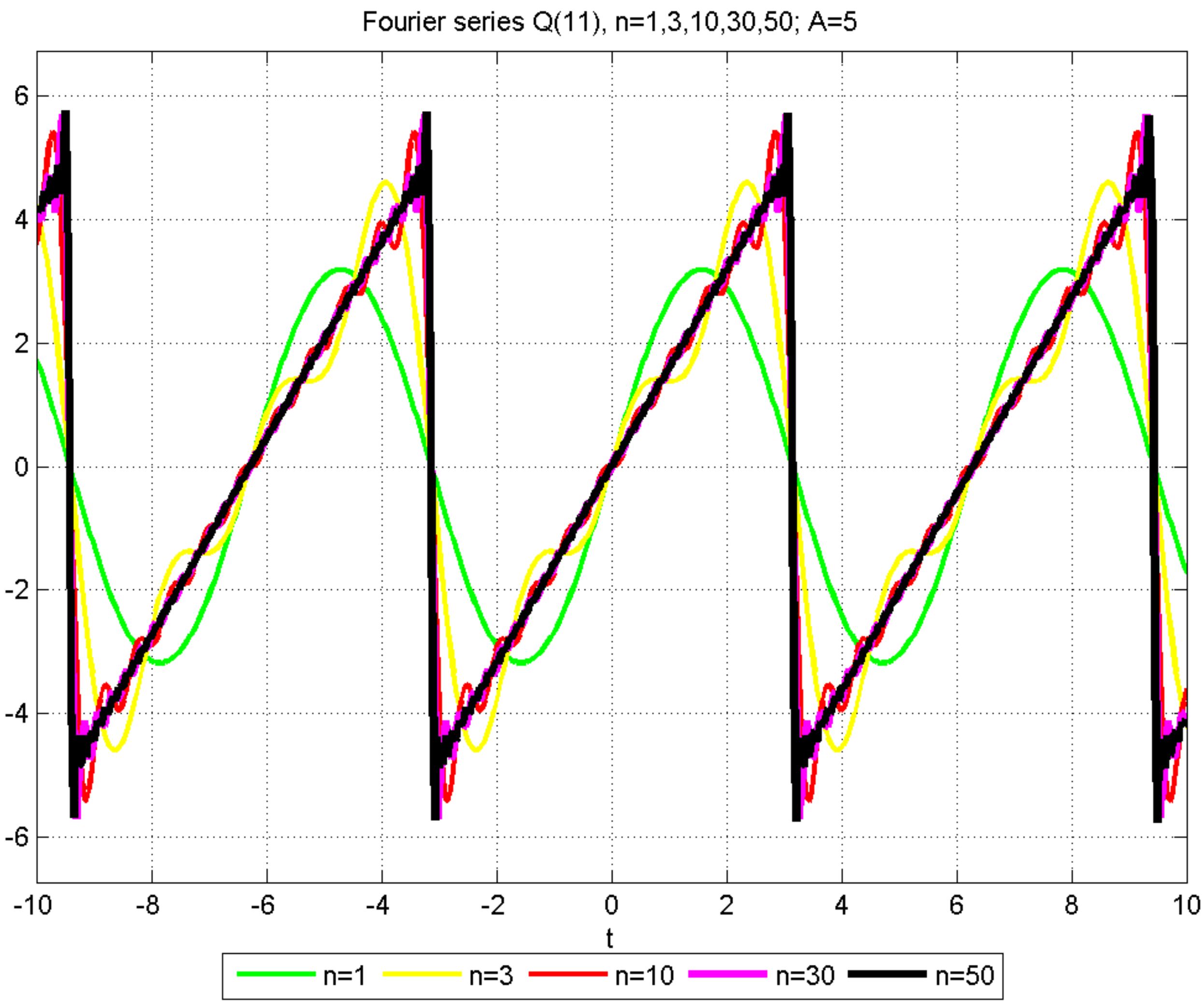
$$b_n = \frac{-2A}{n\pi} \cos(n\pi) \qquad , \cos(n\pi) = (-1)^n$$

$$b_n = \frac{2A}{n\pi} (-1)(-1)^n = \frac{2A}{n\pi} (-1)^{n+1}$$

For $n$ odd $\Rightarrow b_n = \frac{2A}{n\pi}$

For $n$ even $\Rightarrow b_n = \frac{-2A}{n\pi}$

$$S(t) = \frac{2A}{\pi} \left( \sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \ldots \right)$$


Fourier series Q(11), n=1,3,10,30,50; A=5
6
4
2
0
-2
-4
-6
-10
-8
-6
-4
-2
0
2
4
6
8
10
t
n=1
n=3
n=10
n=30
n=50

Q 12:- $s(t) = 4\sin 50\pi t$

ⓐ Direct integration method

For $x(t) = A\cos(\omega t) \Rightarrow P_x = \frac{A^2}{2}$

so $P_x = \frac{16}{2} = 8$ watt

ⓑ The Parseval's theorem

$$P_x = \sum_{n=-\infty}^{\infty} |D_n|^2 = |D_0|^2 + 2\sum_{n=1}^{\infty} |D_n|^2$$

$D_n = \frac{a_n - jb_n}{2}$ , The signal is odd $\Rightarrow D_0 = 0,\ a_n = 0$

$D_n = \frac{-j(4)}{2} = -2j$

$P_x = 2\sum_{n=1}^{\infty} |D_n|^2 = 2 \times |-2j|^2 = 2 \times 4 = 8$ Watt.

Q13:- $f(t) = \frac{2}{T} t$

$$f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin \frac{2\pi n t}{T}$$

$$D_n = \frac{a_n - j b_n}{2}$$

$$b_1 = \frac{2}{\pi} \times \frac{1}{1} = \frac{2}{\pi} \qquad D_1 = \frac{-j \, 2/\pi}{2} = \frac{-j}{\pi}$$

$$b_2 = \frac{2}{2\pi}(-1) = \frac{-1}{\pi} \qquad D_2 = \frac{j/\pi}{2} = \frac{j}{2\pi}$$

$$b_3 = \frac{2}{3\pi}(1) = \frac{2}{3\pi} \qquad D_3 = \frac{-j \, 2/3\pi}{2} = \frac{-j}{3\pi}$$

$$P_x = |D_0|^2 + 2\sum_{n=1}^{\infty} |D_n|^2 = 0 + 2\left[\left|\frac{-j}{\pi}\right|^2 + \left|\frac{j}{2\pi}\right|^2 + \left|\frac{-j}{3\pi}\right|^2\right]$$

$$P_x = 2\left[\frac{1}{\pi^2} + \frac{1}{4\pi^2} + \frac{1}{9\pi^2}\right] = \cancel{2}\left[\frac{36+9+4}{\cancel{36}_{18}\pi^2}\right] = \frac{49}{18\pi^2} \text{ Watt.}$$

Q14:- $f(t) = \begin{cases} 4t, & 0 \leq t \leq 1 \\ -4t, & -1 \leq t \leq 0 \end{cases}$

The signal is even $\Rightarrow b_n = 0$ , $T = 4$ , $\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a_0 = \frac{\text{Area over one } T}{T} = \frac{\frac{1}{2} \times 1 \times 4 + \frac{1}{2} \times 1 \times 4}{4} = \frac{4}{4} = 1$$

The signal is even, we can evaluate from $0 \to 1$ and multiply by 2

$$a_n = \frac{4}{T} \int_0^1 4t \cos(n\omega t) \, dt$$

$u = t \qquad dv = \cos(n\omega t)\,dt$

$du = dt \qquad v = \frac{\sin(n\omega t)}{n\omega}$

$$a_n = \frac{4}{4} \int_0^1 t \cos(n\omega t) \, dt$$

$$a_n = \frac{4}{n\omega}\left[t \sin(n\omega t)\Big|_0^1 - \int_0^1 \sin(n\omega t) \, dt\right]$$

$$a_n = \frac{4}{n\omega}\left[t \sin(n\omega t)\Big|_0^1 + \frac{\cos(n\omega t)}{n\omega}\Big|_0^1\right]$$

$$a_n = \frac{4}{n\omega}\left[\sin(n\omega) - 0 + \frac{\cos(n\omega)}{n\omega} - \frac{\cos(0)}{n\omega}\right]$$

$$a_n = \frac{4}{n\frac{\pi}{2}}\left[\frac{\sin\left(\frac{n\pi}{2}\right)+\cos\left(\frac{n\pi}{2}\right)}{n\frac{\pi}{2}} - \frac{1}{\frac{n\pi}{2}}\right]$$

$$a_n = \frac{8}{n\pi}\left[\sin\left(\frac{n\pi}{2}\right) - \frac{2}{n\pi} + \frac{\cos\left(\frac{n\pi}{2}\right)}{n\frac{\pi}{2}}\right]$$

$\sin\left(\frac{n\pi}{2}\right) = 1$ for $n = 1, 5, 9, \ldots$

$\sin\left(\frac{n\pi}{2}\right) = -1$ for $n = 3, 7, 11, \ldots$

$\sin\left(\frac{n\pi}{2}\right) = 0$ for $n$ even

$\cos\left(\frac{n\pi}{2}\right) = 0$ for $n$ odd.

$\cos\left(\frac{n\pi}{2}\right) = -1$ for $n = 2, 6, 10, \ldots$

$\cos\left(\frac{n\pi}{2}\right) = 1$ for $n = 4, 8, 12, \ldots$

$$a_1 = \frac{8}{\pi}\left[1 - \frac{2}{\pi}\right]$$

$$a_2 = \frac{4}{\pi}\left[-\frac{2}{2\pi} - \frac{2}{2\pi}\right] = \frac{4}{\pi}\left[-\frac{2}{\pi}\right] = \frac{-8}{\pi^2}$$

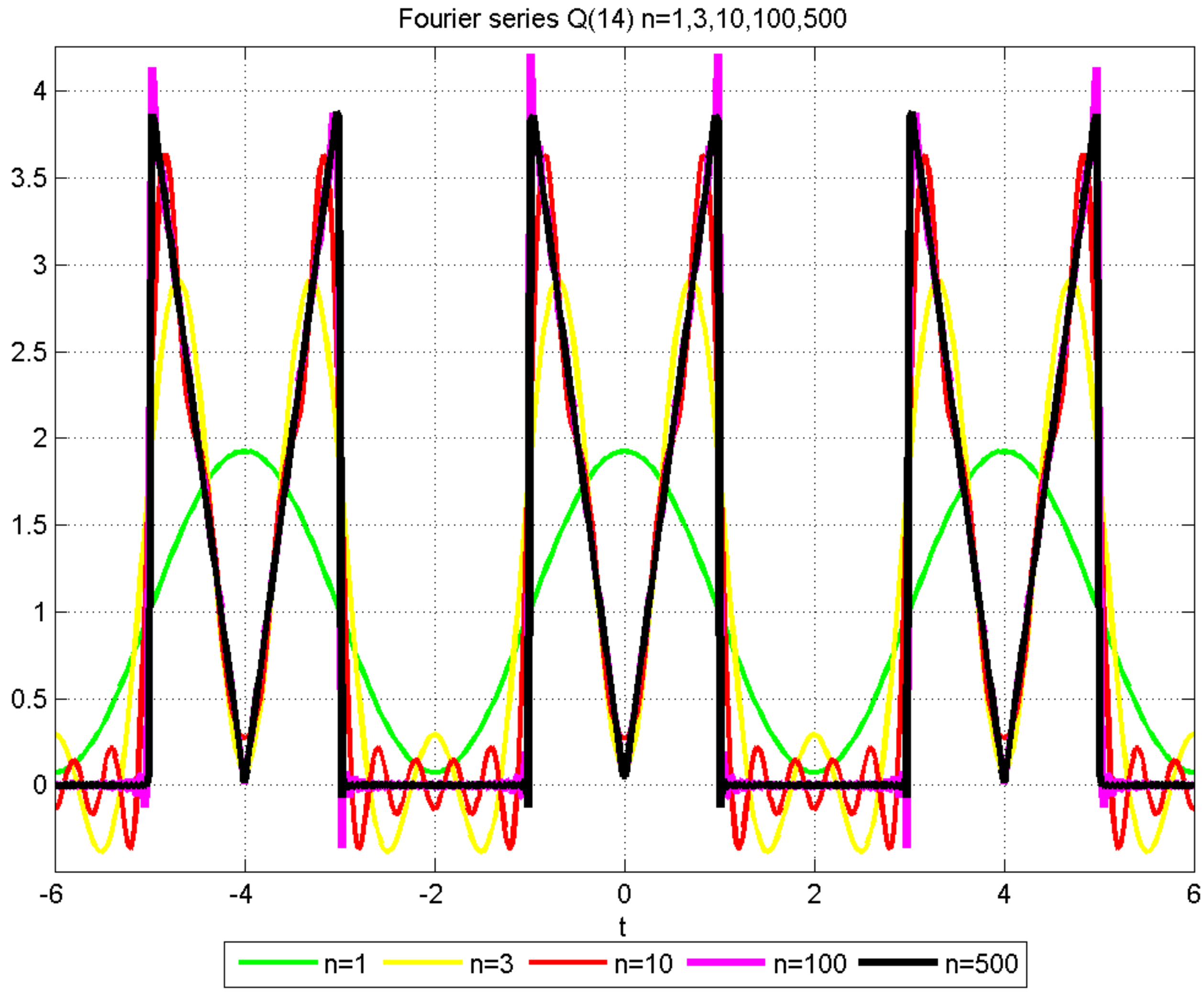
$$a_3 = \frac{8}{3\pi}\left[-1 - \frac{2}{3\pi}\right]$$

$$a_4 = \frac{2}{\pi}\left[\frac{-2}{4\pi} + \frac{2}{4\pi}\right] = 0$$

$$a_5 = \frac{8}{5\pi}\left[1 - \frac{2}{5\pi}\right]$$

$$f(t) = 1 + \frac{8}{\pi}\left(1 - \frac{2}{\pi}\right)\cos(\omega t) - \frac{8}{\pi^2}\cos(2\omega t) + \frac{8}{3\pi}\left(-1 - \frac{2}{3\pi}\right)\cos(3\omega t)$$

$$+ \frac{8}{5\pi}\left(1 - \frac{2}{5\pi}\right)\cos(5\omega t) + \ldots$$



Fourier series Q(14) n=1,3,10,100,500
4
3.5
3
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
t
n=1
n=3
n=10
n=100
n=500